Greater Richmond Council of Teachers of Mathematics

The Great Circle

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GRCTM Spring Conference 2007

Focus on the Future

The theme of the GRCTM Spring Conference will be ***Focus on the Future***. We are seeking speakers who can present lessons that connect to 21st century skills, as we prepare our students to be successful in this age. All session ideas will be considered and we need all levels K-12+. Please consider submitting a speaker proposal and encouraging your colleagues to do the same. Simply fill out the speaker form available on our website **www.grctm.soe.vcu.edu** and email, fax or snail mail to bdavis@mathsciencecenter.info.

The GRCTM Spring Conference will be held on Tuesday evening, April 17, 2007, at Manchester Middle School in Chesterfield County. The school is located at 7401 Hull Street, which is Route 360 convenient to Chippenham Parkway.

Our board has agreed that all who have paid their membership up through the spring conference will have their membership extended through our next school year 2007-08, so this is a great time to recruit new members. Invite them to the Spring Conference and let them know that paid membership will include 1 ½ years of full membership. We love fractions!

Betsey Davis
Conference Chairperson

A Mathematical Quickie[1]

On April 1, 1946, the *Erewhon Daily Howler* carried the following item: "The famous astrologer and numerologist of Guayazuela, the Professor Euclide Paracelso Bombasto Umbugio, predicts the end of the world for the year 2141. His prediction is based on profound mathematical and historical investigations. Professor Umbugio computed the value of the formula

$$1492^n - 1770^n - 1863^n + 2141^n$$

for $n = 0, 1, 2, 3$, and so on, up to 1945, and found that all the numbers which he obtained in many months of laborious computation are divisible by 1946. Now, the numbers 1492, 1770, and 1863 represent memorable dates: The Discovery of the New World, the Boston Massacre, and the Gettysburg Address. What important date may 2141 be? That of the end of the world, obviously."

Deflate the professor! Obtain his result with little computation. [See p. 2.]

[1] Charles W. Trigg, *Mathematical Quickies*, Dover Publications, ©1967, 1985, #23, p. 8.

President's Corner

As a mathematics coordinator, I spend a lot of my time at work walking through the halls of several schools from elementary through high school. My goal is to observe math instruction wherever it is happening to see not only what the teachers are doing, but how the students are responding. I find it very interesting (and sad) to find that both the teachers and the students become more sedentary from the middle grades on. It really seems to begin to occur around the 4th or 5th grade. It's not just one teacher, either; almost everyone is doing this in almost every core subject. As a teacher, I never sat in the classroom unless I needed to be out of the way for a student presentation in front of the class. Even when the students were taking a test, I found myself wandering about the classroom to see what the students were doing. Research tells us very clearly that the brain cannot focus on new instruction for more than 12-15 minutes at a time, yet I see teachers lecturing for anywhere from 30 to 60 minutes with little to no time to process. Teachers are so focused on "covering the curriculum" that they have lost sight of what really matters with instruction and what really works with instruction. It only takes a quick visit to an elementary classroom to see both the students and the teachers moving about almost constantly to see what really works. No wonder students lose their interest and excitement in learning math by the time they get through middle school. They have been tortured with worksheets and other seat work, repetition without connection, in the same manner that teaching was done 20 years ago. The real problem is that today's student needs more stimulation, having grown up in the age of technology that is full of stimulation. Middle school and high school teachers need to look at different strategies for instruction. Even sending a teacher to a conference to get some new ideas does not always do the trick, especially when the workshop sessions for their grade level are often presented the same way – through lecture. I have found that the best sessions for getting ideas are the elementary sessions. They are most often high energy and they have some great activities to share that can be adapted to use in the middle school or high school classroom. Those of you who are middle school or high school teachers might want to try it out. The next time you attend a GRCTM conference (April 17) or a VCTM Conference (March 9-10), take in at least one elementary math session that looks like it might offer some interesting strategies for teaching. You will be surprised at what you can learn from the experience. And if you are not able to do that, then I would suggest that you read *Classroom Instruction That Works* by Marzano, Pickering and Pollack. It's time to energize our students to learn and apply mathematics so that they will be able to be successful in this increasingly more competitive world market.

Respectfully,
Diane Leighty
GRCTM President, 2006 – 07

ANSWER TO THE MATHEMATICAL QUICKIE ON P. 1

That $x - y$ is a divisor of $x^n - y^n$ for $n = 0,1,2,\ldots,$ is the only principle required. Let the professor's number be $F(n) = 1492^n - 1770^n - (1863^n - 2141^n)$ Then since 2141 – 1863 = 1770 – 1492 = 278, $F(n)$ is always divisible by 278. Similarly, 2141 – 1770 = 1863 – 1492 = 371, which is relatively prime to 278. So $F(n)$ is always divisible by (278)(371) = (53)(1946), and hence, of course, by 1946 itself.

Euler at 300

This year marks the 300th anniversary of the birth of Leonhard Euler (pronounced *Oiler*), one of the greatest mathematicians ever, and certainly the most prolific. Euler was Swiss and studied at the University of Basel, where he was strongly influenced by one of the famed Bernouilli brothers. He wrote about it himself, "... I soon found an opportunity to be introduced to a famous professor Johann Bernoulli. ... True, he was very busy and so refused flatly to give me private lessons; but he gave me much more valuable advice to start reading more difficult mathematical books on my own and to study them as diligently as I could; if I came across some obstacle or difficulty, I was given permission to visit him freely every Sunday afternoon and he kindly explained to me everything I could not understand." Euler completed his studies at Basel in 1726 and took up a position at St. Petersburg, Russia, with Daniel Bernoulli already there. He spent the rest of his life in Russia or Prussia as a court mathematician, producing a prodigious amount of mathematics. It is said that if all his mathematics were collected in one place it would occupy 60,000 quarto size pages in print. Since it is often said that mathematics is the province of the young, it should be noted that over half of Euler's works were produced after he was 59 when he became totally blind. He lived to be 76.

We owe to Euler many of the symbols we are most familiar with including π, e, and i, and the remarkable equation that connects them

$$e^{i\pi} + 1 = 0 .$$

"We may safely say, that the whole form of modern mathematical thinking was created by Euler. It is only with the greatest difficulty that one is able to follow the writings of any author immediately preceding Euler, because it was not yet known how to let the formulas speak for themselves. This art Euler was the first one to teach."—F. Rudio, Quoted by Ahrens W.: *Scherz und Ernst in der Mathematik* (*Leipzig, 1904*), p. 251.[1]

"The general knowledge of our author [Leonhard Euler] was more extensive than could well be expected, in one who had pursued, with such unremitting ardor, mathematics and astronomy as his favorite studies. He had made a very considerable progress in medical, botanical, and chemical science. What was still more extraordinary, he was an excellent scholar, and possessed in a high degree what is generally called erudition. He had attentively read the most eminent writers of ancient Rome; the civil and literary history of all ages and all nations was familiar to him; and foreigners, who were only acquainted with his works, were astonished to find in the conversation of a man, whose long life seemed solely occupied in mathematical and physical researches and discoveries, such an extensive acquaintance with the most interesting branches of literature. In this respect, no doubt, he was much indebted to an uncommon memory, which seemed to retain every idea that was conveyed to it, either from reading or from meditation."—C. Hutton, *Philosophical and Mathematical Dictionary* (*London, 1815*), *pp. 493-494.*[2]

[1] R.E. Moritz, *Memorabilia Mathematica*, Math. Asso. of America, ©1914, 1942, #957, p. 154.
[2] *op. cit.*, #958, p154.

Card Games

The following card games are played with a deck of cards with the face cards removed.

Evens and Odds

Players: 2 – 4

Directions: Shuffle and stack the cards face down in a pile.

The first player guesses whether the top card in the pile is even or odd, then turns it face up. If the guess is correct, the player keeps the card and takes another turn. If the guess is incorrect, the card is placed in the discard pile. The player continues to guess and turn cards over until a guess is incorrect. Players take turns until there are no more cards in the pile. The player with the most cards is the winner!

Find Your Place Value

2 – 4 players Remove tens as well as the face cards Two decks of cards can be used for more than 2 players.

Directions: Deal out the cards evenly to all players. Players places their cards facedown in front of them. Each player turns over 3 cards, then arranges the cards to make the greatest possible three- digit number. Players read their numbers. The player with the greatest number wins all of the cards from that round and places them in a separate pile. Play continues until all of the cards have been used. The player with the most cards at the end of the game is the winner!

Players can use four or more cards to work with larger numbers. Players can also form the lowest possible number to win a round.

A Game, More or Less

Players: 2

Directions: Deal out the cards evenly to both players. Players stack their cards facedown in front of them. Both players turn over their top card. Player 1 compares the value of the two cards by answering these questions:

Is mine more or less?

How many more? or How many less?

The player with the greater value card wins both cards and puts them in a separate stack. If the cards are equal, players draw again and the winner takes both pairs of cards. Players continue to take turns until all cng to have the player with the lesser-value card to win the cards.

Take 10!

Players: 2

Directions: One player stacks the cards facedown in a pile.

Players decide who will collect cards that are 'Less than 10 and who will collect cards that are "10 or more." Player 1 draws two cards and adds the numbers on the cards, saying the equation. (ex. 7 plus 5 equals 12) If the sum of the numbers is less than 10, "the less than 10" player wins the cards. If the sum is 10 or more, the "10 or more" player wins the cards. Players take turns until all of the cards have been played. The player with the most cards is the winner!

Go for 10!

Players: 2

Directions: Deal 10 cards to each player. Set the timer for 3 minutes. Players turn their cards over at the same time. During the 3 minutes, players use their cards to make as many addition, subtraction, or multiplication problems as they can that equal 10. Each card can only be used once. A 10 card alone can equal 10. When time is up, players tell each other the equations they made. Players keep the cards used in their equations and return the unused cards to the deck. The game continues until all cards have been used. The final round will be played with fewer than 10 cards per player. The player with the most cards is the winner!

Calculating Cards

Players: 2 or more. Materials: paper, pencil, calculator

Directions: Deal four cards to each player. Stack the rest of the cards facedown in a pile. Turn over the top two cards and place them side by side. The two cards together form a 2-digit "goal number." The first card represents the tens place and the second card represents the ones place. A player reads aloud the goal number. Each player arranges his or her cards to form two 2-digit numbers. The object is to make an addition or subtraction problem with an answer as close to the goal number as possible. The player with the closest answer scores 1 point. Place the used cards in a discard pile. The next round begins by drawing two cards to form a new goal number. Play continues in the same way. The game ends when all the cards have been played. The player with the most points is the winner!

Students can play using three or four-digit numbers.

Balancing Act

Players: 2 – 4

Materials: Balancing Act Game Sheet

Directions: Deal 6 cards to each player and stack the rest of the cards face down in a pile. Each player chooses 4 cards from his or her hand to place on the game sheet. The object is to balance the scale by arranging th cards into two addition problems with equal sums. A player earns 1 point for balancing the scale. Ex. A player could place 6 + 2 on one side and 4 + 4 on the other side. A player can also place the same card on both sides to balance the scale. (5 + 0 = 5 + 0) The empty space represents 0. At the end of a round, the cards played are placed at the bottom of the deck. Shuffle the cards and give each player 6 more cards. Continue playing the same way. The game ends when one player earns 10 points (or another predetermined amount).

* This game could be varied to include subtraction, multiplication, and division.

Solitary

Players: 1

Directions: The player arranges nine cards face up in three rows of three, and stacks the rest of the cards facedown in a pile. From the nine cards the player picks up 2 or more cards with a sum of 11. Then fill the spaces with cards from the pile. If there are no cards that add up to 11, add another row of three cards from the pile. Continue to pick up cards that total 11. The game ends when all of the cards from the pile have been used and no cards remain whose numbers add up to 11. The remaining cards can be counted and recorded. Then play again to try to have fewer remaining cards.

- Students can use other numbers as a total, such as 12, 13, 14, etc.

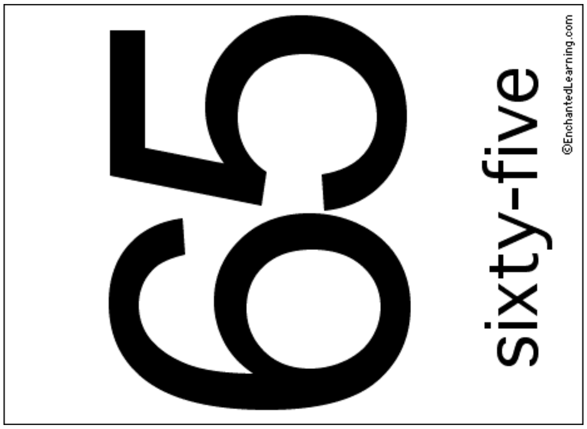
Joan Stoller
Title I Math
Highland Springs Elementary School

A Card Trick[1]

Use only the kings and queens from the deck. The kings are placed in one pile, the queens in another. The piles are turned face down and one placed on top of the other. A spectator cuts the packet of eight cards as often as desired. The magician holds the packet out of sight. In a moment the magician brings forward a pair of cards and tosses them face up on the table. They prove to be a king and queen of the same suit. This is repeated with the other three pair.

Method: When the two piles are formed, the magician makes sure that the order of suits is the same in each pile. Cutting will not disturb this rotation of suits. Out of sight, the magician simply divides the packet in half, then obtains each pair by taking the top card of each half. These two cards will always be a king and queen of the some suit.

[1] Martin Gardner, *Mathematics Magic & Mystery*, Dover Publications, ©1956, p. 15.



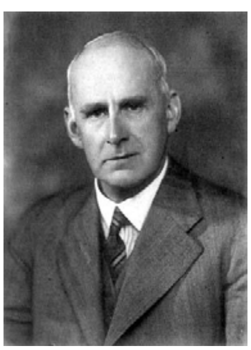
Sixty-five is the sum of two squares in two different ways:[2]

65 = $8^2 + 1^2$ and **65** = $7^2 + 4^2$.

3	7	14	16	25
11	20	23	2	9
22	4	6	15	18
10	13	17	24	1
19	21	5	8	12

[3]

65 is part of three Pythagorean triples[4], (16, 63, 65), (33, 56, 65) and (65, 72, 97). Therefore, **65** is the smallest hypotenuse of two different right triangles that have sides that are natural numbers, since $65^2 = 63^2 + 16^2$ and also $65^2 = 56^2 + 33^2$.



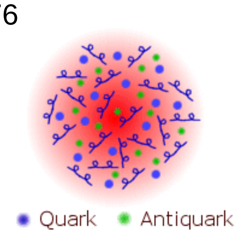
Sir Arthur Eddington[5] (1882 – 1944) claimed, on the basis of physical theory, that there are *exactly*

$$17 \times 2^{259}$$

protons in the universe. That is, he claimed that in the universe there are

15,747,724,136,275,002,577,605,653,961,181,555,468,044,717,914, 527,116,709,336,231,425076,185,631,031,276

protons—accurate to the last digit!



[2] David Wells, *The Penguin Dictionary of Curious and Interesting Numbers*, Penguin Books, ©1986, p. 128.
[3] http://www.geocities.com/~harveyh/magicsquare.htm
[4] Bryan Bunch, *The Kingdom of Infinite Number*, W. H. Freeman & Co., ©2000, p. 159.
[5] Howard Eves, *Mathematical Circle Revisited*, Prindle, Weber & Schmidt, ©1971, 30°, p. 23.

Teaching the Metric System for America's Future

A Position of the National Council of Teachers of Mathematics

Question: Why should schools teach the metric and customary systems of measurement?

NCTM Position

To equip students to deal with diverse situations in science and other subject areas, and to prepare them for life in a global society, schools should provide students with rich experiences in working with both the metric and the customary systems of measurement while developing their ability to solve problems in either system.

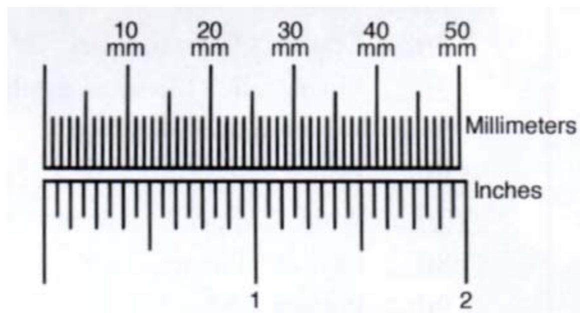
Teaching Measurement

Making measurements and using them effectively are essential life skills. Measuring with standard units in one, two, or three dimensions helps quantify the world. Students need to understand how to assign numbers to attributes such as length, area, volume, weight, and temperature and know how to determine accurate measurements in units appropriate to the context and attribute. The best way to teach measurement is to have students estimate and then measure particular attributes, beginning with everyday items and personal points of reference.

Estimating and measuring are only part of what students need to understand and be able to do in measurement. They must develop fluencies in translating among related measurements within a system and in using proportional relationships. They also must understand that although precision is important, all measurements are approximations. Students need to be able to solve problems that involve measurement.

The Metric and Customary Systems of Measurement

In today's global environment, metric measurements are prominent in workplaces, consumer products, and news reports. Almost every other country in the world uses the metric system of measurement. The European Union, Japan, and Korea have passed legislation limiting international commerce to products measured in metric units. If the United States is to continue to play a leading role in international business, using metric measurement is imperative and U.S. workers at all levels must be knowledgeable about the *Système Internationale (SI),* the international name for the metric system. In the United States, many products using metric units are part of everyday work. Instruction can relate the metric system directly to our number system. Units are based on powers of ten, providing instructional opportunities for students to use decimals in applied settings.



The United States continues to make progress in implementing metric measurement in more goods and services. The National Council of Teachers of Mathematics supports efforts by the U.S. government to make a transition to the metric system (*SI*) as the nation's primary measurement system and to reestablish the U.S. Metric Board to support and encourage the use of the metric system. However, the Council recognizes the leadership responsibility of schools to ensure that all students have experiences that enable them to measure in both the metric and the customary systems as well as to solve problems related to measurement in either system.

(October 2006)

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Greater Richmond Council of Teachers of Mathematics
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